## Exercise 35

The unemployment rate $U(t)$ varies with time. The table gives the percentage of unemployed in the US labor force from 2003 to 2012.
(a) What is the meaning of $U^{\prime}(t)$ ? What are its units?
(b) Construct a table of estimated values for $U^{\prime}(t)$.

| $t$ | $U(t)$ | $t$ | $U(t)$ |
| :---: | :---: | :---: | :---: |
| 2003 | 6.0 | 2008 | 5.8 |
| 2004 | 5.5 | 2009 | 9.3 |
| 2005 | 5.1 | 2010 | 9.6 |
| 2006 | 4.6 | 2011 | 8.9 |
| 2007 | 4.6 | 2012 | 8.1 |

Source: US Bureau of Labor Statistics

## Solution

$U^{\prime}(t)$ is the rate at which the percentage of unemployed people is increasing with respect to time (units of \%/year). To obtain the values of $U^{\prime}(t)$, calculate the slope of the secant line going through two adjacent $t$ values. At $t=2003$, for example,

$$
U^{\prime}(t)=\frac{U(2004)-U(2003)}{2004-2003}=\frac{5.5-6.0}{1}=-0.50 .
$$

At $t=2004$, there are two secant lines.

$$
\begin{aligned}
& U^{\prime}(t)=\frac{U(2004)-U(2003)}{2004-2003}=\frac{5.5-6.0}{1}=-0.50 \\
& U^{\prime}(t)=\frac{U(2005)-U(2004)}{2005-2004}=\frac{5.1-5.5}{1}=-0.40
\end{aligned}
$$

At such times where there are two possible secant lines, take the average for the best estimate.

$$
\frac{(-0.5)+(-0.4)}{2}=-0.45
$$

| $t$ | $U(t)$ | $U^{\prime}(t)$ |
| :---: | :---: | ---: |
| 2003 | 6.0 | -0.50 |
| 2004 | 5.5 | -0.45 |
| 2005 | 5.1 | -0.45 |
| 2006 | 4.6 | -0.25 |
| 2007 | 4.6 | 0.60 |
| 2008 | 5.8 | 2.35 |
| 2009 | 9.3 | 1.90 |
| 2010 | 9.6 | -0.20 |
| 2011 | 8.9 | -0.75 |
| 2012 | 8.1 | -0.80 |

